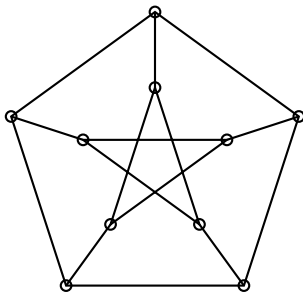


# Tutorial 1

## Advanced Graph Theory

**Degree of a Vertex:** The degree of a vertex is defined as the number of edges incident on it.

*What about loop edges?*



1. Prove or disprove:

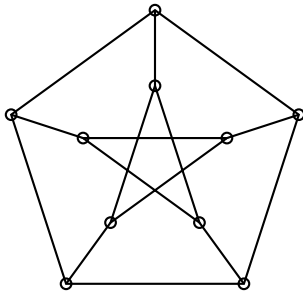
- 1.1 If every vertex of a simple graph  $G$  has a degree 2, then  $G$  is a cycle.
- 1.2 If every vertex of a connected simple graph  $G$  has a degree 2, then  $G$  is a cycle.
- 1.3 The complement of a simple disconnected graph must be connected.

2. Prove by induction, that the cycle  $C_n$ ;  $n = 2k + 1, k \geq 0$ , can never be a sub-graph of  $K_{r,s}$ .

### 3. Definition

#### *Petersen Graph*

The **Petersen graph** is the simple graph whose vertices are the 2-element subsets of a 5-element set and whose edges are the pairs of disjoint 2-element subsets.



Determine whether the Petersen graph is bipartite, and find the size of its largest independent set. (Hint: Use the results learned in questions already solved) Why do you say that it is the largest independent set?

4. Given a connected simple graph  $G$  and  $v \in V(G)$ ; prove that  $v$  has a neighbor in every component of  $G - v$ . Can you then conclude that no graph has a cut-vertex of degree 1?
5. Let  $W$  be a closed walk of length atleast 1 **that does not contain a cycle**. Prove that some edge of  $W$  repeats immediately (once in each direction).
6. Let  $v$  be a cut-vertex of a simple graph  $G$ . Prove that  $\bar{G} - v$  is connected.